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Flow of a Circular Jet into a Cross Flow

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Nomenclature

C_D = jet cross-flow drag coefficient
d_0 = jet exit diameter
E = jet entrainment parameter
$g(m)$ = arbitrary function of m
m = U_j/U
t = time
U = cross-flow velocity
U_j = jet exit velocity
X,Z = jet centerline coordinates
α,β,γ = exponents in Eq. (4)

In recent years there has been considerable interest in the aerodynamics of aircraft having the capability of vertical or short takeoff and landing (V/STOL). One method for achieving V/STOL is to use lift engines or fans so that an important aerodynamic problem is to determine the flowfield due to a jet exhausting at right angles into a uniform cross flow. A number of methods for predicting this flowfield have been put forward.[1–5] In the majority of these methods it is necessary to know the position of the jet centerline, and empirical formulae have been used to define the jet path. When a detailed analysis of the jet cross-flow interference problem is made, it is usually not possible to obtain an expression for the jet centerline in closed form.

One expression for the jet centerline has been determined in Ref. 6. This expression shown in Eq. (1) was obtained by considering jet entrainment of cross-flow fluid and the blockage effect that the jet has on the cross flow.

$$\frac{X}{d_0} = \frac{\pi(C_D + 2E)}{8E^2}\left[\exp\left(\frac{4E}{\pi}\cdot\frac{Z}{md_0}\right) - \frac{4E}{\pi}\cdot\frac{Z}{md_0} - 1\right] \quad (1)$$

The expression of Eq. (1), valid for moderate values of m, has the interesting property that it implies X/d_0 = function (Z/md_0). It is the purpose of this note to show that this functional relationship is valid for large values of m.

Let us consider the flow of a circular jet exhausting at right angles from a plane wall into a uniform cross-flow. If $m \gg 1$, the three-dimensional steady problem may be investigated by considering a related two-dimensional unsteady problem. In this case at time $t = 0$, the jet is replaced by a distribution of line vortices perpendicular to the wall. The strengths of these vortices are chosen so as to represent the flow of a uniform stream U around the jet which is represented as a solid boundary. At time $t = 0$ the vortices are allowed to move as free vortices, and their subsequent motion represents the jet deformation under the influence of a cross-flow.

Now t is related to the jet exit velocity U_j and the distance Z perpendicular to the wall by the expression $Z = U_jt$. Also, the displacement of the jet centerline X will be a function of U, t, and d_0, that is,

$$X = \text{function } (U,t,d_0) \quad (2)$$

or

$$X = \text{function } (U, Z/U_j,d_0) \quad (3)$$

Writing this function in series form,

$$X = \sum_{\alpha=-\infty}^{\infty}\sum_{\beta=-\infty}^{\infty}\sum_{\gamma=-\infty}^{\infty} U^{\alpha}\left(\frac{Z}{U_j}\right)^{\beta} d_0^{\gamma} \quad (4)$$

Then dimensional considerations imply that $\beta = \alpha$, $\gamma = \alpha + 1$ so that

$$\frac{X}{d_0} = \sum_{\alpha=-\infty}^{\infty}\left(\frac{Z}{d_0}\cdot\frac{U}{U_j}\right)^{\alpha} \quad (5)$$

Then

$$X/d_0 = \text{function } (Z/md_0) \quad (6)$$

This functional relationship is verified in Table 1 using data obtained from Ref. 7. Plotting the data of Table 1 in the form log X/d_0 vs log(Z/md_0), shown in Fig. 1, it has been possible to find a simple form for the equation of the jet centerline. This equation

$$(X/d_0)^2 = 7(Z/md_0)^5 \quad (7)$$

also may be written in the form

$$X/m^{5/3}d_0 = 7^{1/2}(Z/m^{5/3}d_0)^{5/2} \quad (8)$$

Williams and Wood[5] have pointed out that certain similarity laws will hold if the equation for the jet path can be expressed in the form $X/g(m)d_0$ = function $[Z/g(m)d_0]$, in which $g(m)$

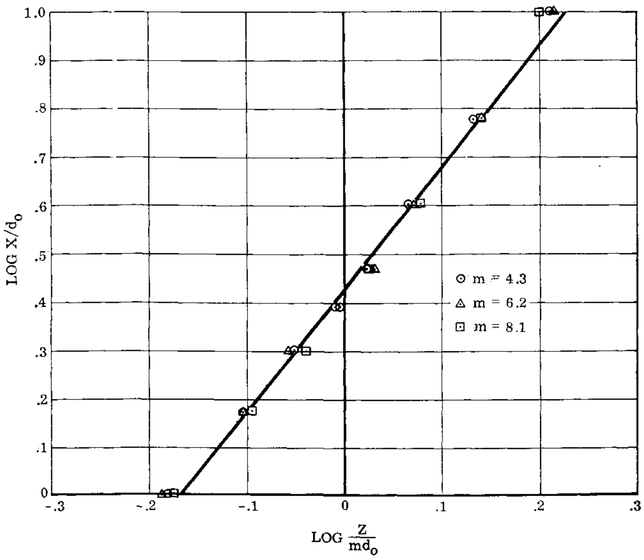

Fig. 1 Correlation of jet centerline data.

Table 1 Tabulation of experimental data (Ref. 7)

X/d_0	m = 4.3	6.2	8.1
	Z/md_0		
1	0.66	0.65	0.67
1.5	0.79	0.79	0.80
2	0.89	0.88	0.91
2.5	0.98	0.99	0.99
3	1.06	1.06	1.07
4	1.17	1.18	1.19
6	1.36	1.39	1.38
10	1.63	1.64	1.60

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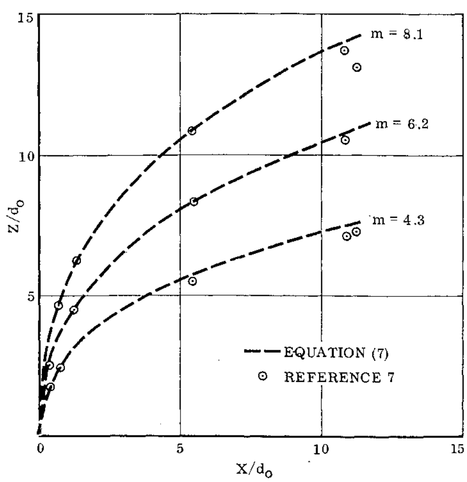

Fig. 2 Jet centerlines.

is any function of m. Equation (8) is seen to be of this form.

Figure 2 shows calculated jet centerlines using Eq. (7). Shown in the same figure are data from Ref. 7, and it is seen that Eq. (7) fits these experimental data quite well. Analysis of other experimental data[8,9] has shown the same functional relationship, although for each set of test data better correlation can be obtained by using slightly different values for the coefficient and exponent of Eq. (7).

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Skin-Friction Formula for Tapered and Delta Wings

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WHEN determining the skin-friction drag for a tapered or delta-shaped wing, it is necessary to take into consideration the variation of the skin-friction coefficient over the whole surface and the dependence on the Reynolds' number spanwise variation with chord length. In principle this can be done in two ways. One method is to choose some equivalent reference chord on the wing for the calculation of the skin-friction coefficient. This method can however give considerable errors. The other method of analysis is to integrate the skin-friction coefficient along the span of the wing, the so-called strip integration method. If the skin-friction formula used is simple enough for the integral to be developed into an explicit expression, the analysis is quite straightforward. Unfortunately, this is not always the case. For the skin friction in turbulent boundary layer, the formulas commonly used will require numerical integration. This can be very time consuming. Much time could be saved if there existed a formula which made the numerical integration unnecessary. The following is such a formula for a tapered surface (one side of the wing) with fully turbulent boundary layer:

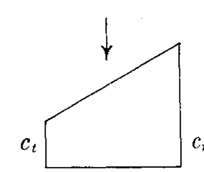
$$c_f = \frac{0.472}{\left({}^{10}\log Re\ \dfrac{1+\lambda}{2}\right)^{2.58}} \cdot \left[1 - \frac{(1-\lambda)^4(4.55 - 0.27\cdot{}^{10}\log Re)}{100}\right]$$

where Re = Reynolds number based on c_r ($c_r > c_t$)

$$\lambda = \frac{c_t}{c_r}$$

c_r = root chord

c_t = tip chord



The formula has been developed by correlating the λ and Re dependence against values calculated by complete integration, using the Prandtl flat-plate turbulent skin-friction formula

$$c_f = 0.472/({}^{10}\log Re)^{2.58}$$

For the Reynolds number range $10^5 - 10^9$, the formula agrees within 0.2% with strip-integrated values.

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